

Pre-class Warm-up

Consider the problem:

Find the entry in row 2 column 3 of the matrix product

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ -1 & 2 & 1 \end{bmatrix}$$

Here's the question I shall ask you today:

Find someone else who can interview about this.

Is that person able to do this calculation?

Answer:

a. Yes

b. No

- There is a 10 minute quiz tomorrow in your discussion session. You may use a single sheet of handwritten notes.
- Have you had problems getting the Canvas site to work? What doesn't work?

2.1

How to understand functions of several variables

$$\text{or } f: D \rightarrow \mathbb{R}^m$$

We learn:

$D \subseteq \mathbb{R}^n$ is an "open" subset.

- How to recognize n and m for a function

$$f: \mathbb{R}^n \rightarrow \mathbb{R}^m \quad \text{D} \xrightarrow{f} \mathbb{R}^m$$

- Domain, range, target or codomain
- Describing a function by its graph
- Describing a function by its level sets, = contour lines.
- Describing a function by sections
- What some standard functions look like: paraboloids, saddle points.

What are n and m in the following functions $f: \mathbb{R}^n \rightarrow \mathbb{R}^m$?

What are the domain and range?

a. $f(s,t) = (1,2,3) + s(0,-1,1) + t(1,0,2)$

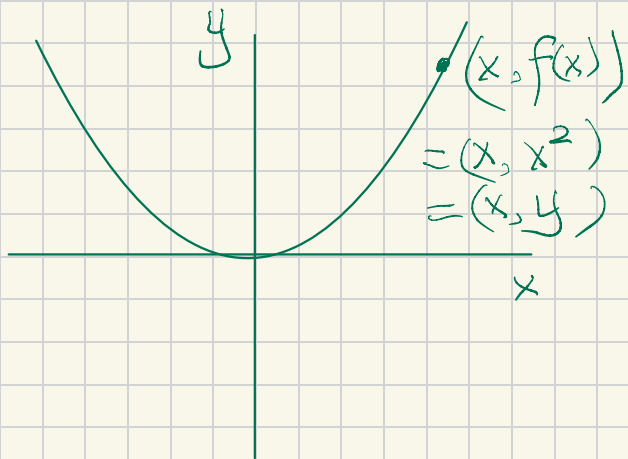
$$n = 2 \quad m = 3$$

b. $f(s) = (1,2,3) + s(0,-1,1)$

- ? $n = 1$ and $m = 1$
- ? $n = 3$ and $m = 1$
- ? $n = 1$ and $m = 3$

The range of f is the line in \mathbb{R}^3 passing through $(1,2,3)$ in direction $(0,-1,1)$ ✓

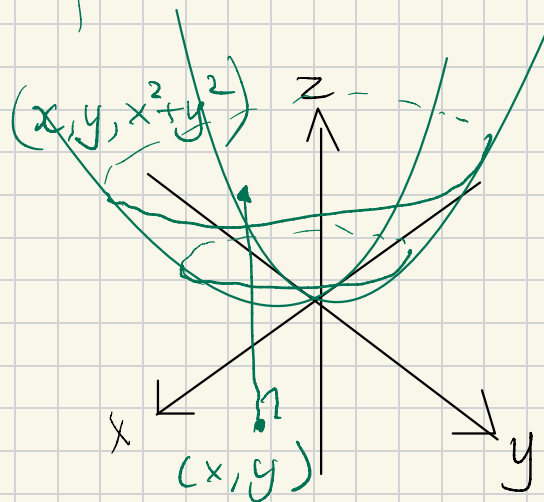
The graph of $f(x) = x^2$.



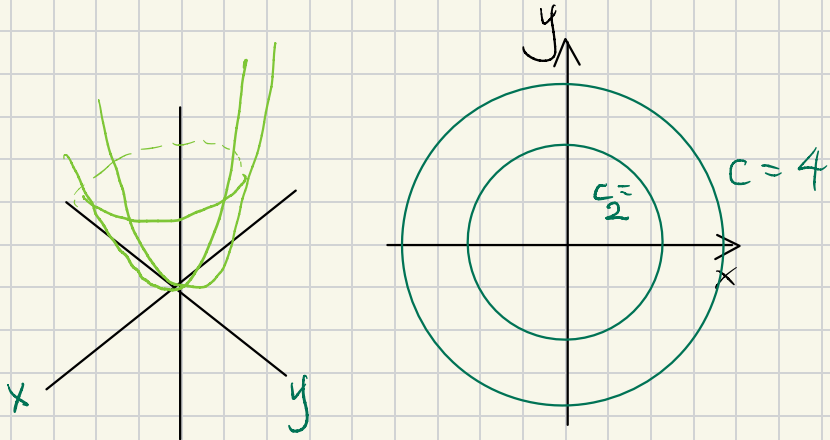
The graph of $f(x,y) = x^2 + y^2 : \mathbb{R}^2 \rightarrow \mathbb{R}$
is the set of points $(x,y, f(x,y))$
in \mathbb{R}^3

When $y = 0$, we get $(x, 0, x^2)$
When $x = 0$ we get $(0, y, y^2)$

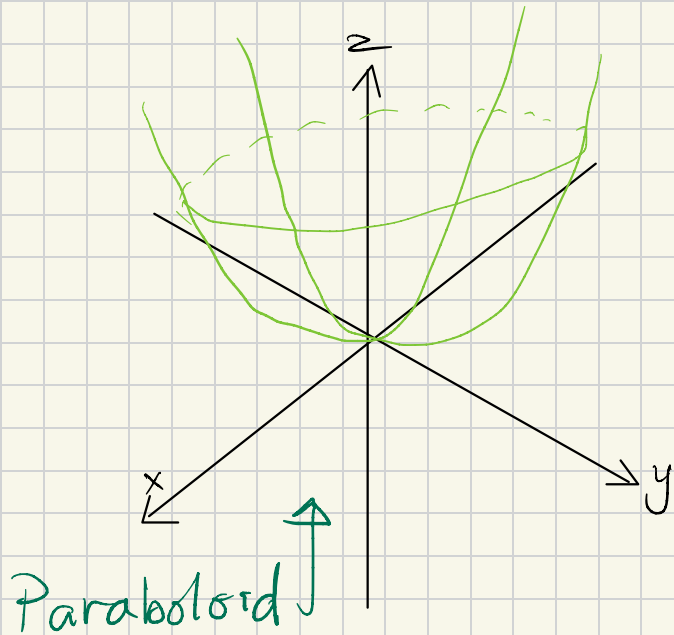
A parabolic dish:



The level sets of $f(x,y) = x^2 + y^2$ are sets in the domain, of points (x,y) where $f(x,y) = c$ is some constant.

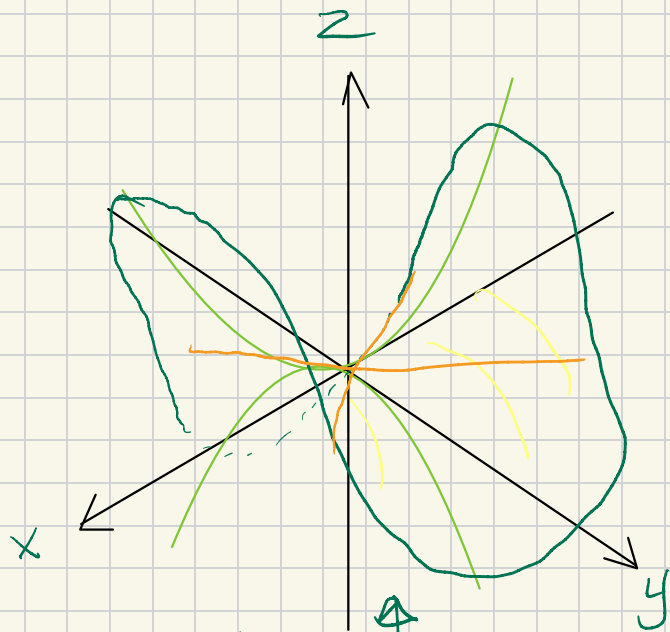


Sections of $f(x,y) = x^2 + y^2$ are intersections of the graph with vertical planes.



$$f(x,y) = x^2 - y^2$$

Sections: parabola pointing up
in $x-z$ plane
parabola pointing down in $y-z$ plane



Saddle point

Level sets

